Mathematics, Physics and Measurement Units

MATHEMATICS

1. Taylor's formula for a function of one variable and its use for approximating functions with polynomials.

Answer: Let $f: I \subseteq R \rightarrow R$ and $x_0 \in I, f \in C_I^{n+1}$. Taylor's Formula states that:

$$f(x) = T_n(x) + R_n(x),$$

where T_n is the Taylor polynomial of n^th degree, and R_n is the remainder:

$$T_n(x) = f(x_0) + \frac{x - x_0}{1!} f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \cdots + \frac{(x - x_0)^n}{n!} f^{(n)}(x_0),$$

$$R_n(x) = \frac{(x - x_0)^{n+1}}{(n+1)!} f^{(n+1)}(x_0 + \theta(x - x_0)), \qquad 0 < \theta < 1.$$

one obtains the following approximation of $f(x)$ in a neighborhood *V* of x_0:

$$f(x) \cong T_n(x),$$

with the error $\varepsilon_n = \sup_{x \in V} |R_n(x)|$.

2. Define the eigenvalues and eigenvectors of a linear operator.

Answer: Let *V* be a vector space over the field *K* and $f: V \rightarrow V$ a linear operator. A non-zero vector $v \in V$ is called an *eigenvector* of the operator *f* if there exists a scalar λ from *K* such that $f(v) = \lambda v$. The scalar λ is called the corresponding *eigenvalue of* v.

3. How does one determine the local extrema of a function of two variables that is partially differentiable?

Answer: The local extrema of the function $u = u(x, y)$ are among the associated stationary points, which are the solutions of the system:

$$\begin{cases} \dfrac{\partial u}{\partial x} = 0 \\ \dfrac{\partial u}{\partial y} = 0. \end{cases}$$

A stationary point is a local minimum point if $\frac{\partial^2 u}{\partial x^2} \cdot \frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 > 0$ and $\frac{\partial^2 u}{\partial x^2} > 0$ at that point, and it is a local maximum point if $\frac{\partial^2 u}{\partial x^2} \cdot \frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 > 0$ and $\frac{\partial^2 u}{\partial x^2} < 0$ at that point.

4. For a discrete random variable, define the following numerical characteristics: the mean value (expectation or expected value), the dispersion (variance) and the standard deviation.

Answer: Let *X* be a discrete random variable with distribution:

$$X: \binom{x_i}{p_i}, i = \overline{1,n}, \sum_{i=1}^{n} p_i = 1, \ \ p_i = P(X = x_i).$$

The mean value or the expectation reprezents a value around which the values of a random variable are grouped. It is computed with the formula:

$$M(X) = \sum_{i=1}^{n} x_i p_i.$$

The dispersion or variance is:

$$D^2(X) = M[(X - M(X))^2].$$

The standard deviation: $D(X) = \sqrt{D^2(X)}$.

The dispersion and the standard deviation measures the ,,scattering" of the values of a random variable around its mean value.

5. Define the Laplace transform of an original function and write the Theorem of differentiation of the original function.

Answer: If *f* is an original function, its *Laplace a transform* is:

$$(\mathcal{L}f(t))(p) = \int_0^{\infty} f(t)e^{-pt}dt.$$

The theorem of differentiation of the original function states that:
If *f* and *f'* are original functions, then

$$(\mathcal{L}f'(t))(p) = p(\mathcal{L}f(t))(p) - f(0_+).$$

6. Define the *Z transform (the discrete Laplace transform*) of an original sequence and compute its image on the discrete unit step signal.

Answer: If $\{f_n\}_n$ is an original sequence, its *Z transform* is:

$$Z(f_n)(z) = \sum_{i=1}^{\infty} f_n z^{-n}.$$

For the discrete unit step signal

$$\sigma(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0, \end{cases} \qquad n \in Z,$$

the Z transform is:

$$Z\sigma(n)(z) = \sum_{i=1}^{\infty} z^{-n} = \frac{1}{1 - \frac{1}{z}} = \frac{z}{z-1},$$

for $|z| > 1$.

7. Polar, cylindric and spherical coordinates.

Answer: *Passing to the polar coordinates:*

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta, \end{cases}$$

where $\rho \in [0,\infty), \theta \in [0,2\pi)$ establishes the connection between the cartesian coordinates *(x,y)* of a point in the plane and its polar coordinates (ρ,θ).

Passing to the cylindric coordinates:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z, \end{cases}$$

where $\rho \in [0,\infty), \theta \in [0,2\pi), z \in R$ establishes the connection between the cartesian coordinates *(x,y,z)* of a point in the space and its cylindric coordinates (ρ,θ,z) .

Passing to the spherical coordinates:

$$\begin{cases} x = \rho\cos\theta\sin\varphi \\ y = \rho\sin\theta\ \sin\varphi \\ z = \rho\ \cos\varphi, \end{cases}$$

where $\rho \in [0,\infty), \theta \in [0,2\pi), \varphi \in [0,\pi)$ establishes the connection between the cartesian coordinates *(x,y,z)* of a point in the space and its spherical coordinates (ρ,θ,φ).

8. Write the Fourier series and the Fourier coefficients of a periodic continuous signal.

Answer: Let $f: R \to R$ be a T-periodic function and let $\omega = \frac{2\pi}{T}$ be the pulsation. The Fourier coefficients are:

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega\, dt,\ \ n = 0,1,\dots$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega\, dt,\ \ n = 1,2,\dots$$

The Fourier series associated to *f* is:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n\sin n\omega t)$$

9. Check that $B = \left\{v_1 = \left(-\frac{1}{3},\frac{2}{3},\frac{2}{3}\right), v_2 = \left(\frac{2}{3},-\frac{1}{3},\frac{2}{3}\right), v_3 = \left(\frac{2}{3},\frac{2}{3},-\frac{1}{3}\right)\right\}$ is a orthonormal basis in the euclidian space R^3 endowed with the standard dot product. Determine the coordinates of the vector $v = (1,2,3)$ in this basis. Write the transition matrix from the standard basis to the basis *B* and compute the eigenvalues and eigenvectors of this matrix.

Answer: First, we prove the the given vectors are two by two orthogonal:

$$\langle v_1, v_2\rangle = -\frac{1}{3}\frac{2}{3} + \frac{2}{3}\left(-\frac{1}{3}\right) + \frac{2}{3}\frac{2}{3} = 0$$

hence $v_1 \perp v_2$. Similarly one checks that $v_2 \perp v_3$ and $v_1 \perp v_3$. Moreover, the norm of each vector is 1, for example

$$\|v_1\| = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1,$$

Hence *B* is an orthonormal basis.

The coordinates of vector *v* in the basis *B* are $\alpha_k = \langle v, v_k\rangle,\ k = 1,2,3$:

$$\alpha_1 = -\frac{1}{3}\cdot 1 + 2\cdot\frac{2}{3} + 3\cdot\frac{2}{3} = 3, \alpha_2 = 2, \alpha_1 = 1,$$

hence $v = 3v_1 + 2v_2 + 1v_3$.
The transition matrix from the standard basis to the basis *B* is:

$$T = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}.$$

The eigenvalues of *T* are the solutions of the characteristic equation $\det(T - \lambda I_3) = 0$, i.e.:

$$\begin{vmatrix} -\frac{1}{3} - \lambda & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} - \lambda & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} - \lambda \end{vmatrix} = 0.$$

One obtains the solutions with multiplicities $\lambda_1 = -1, m_1 = 2$ and $\lambda_2 = 1, m_2 = 1$.
The eigenvectors corresponding to the eigenvalue λ_1 are the solutions of the homogeneous linear system $(T - \lambda I_3)v = 0$:

$$\begin{cases} \frac{2}{3}x + \frac{2}{3}y + \frac{2}{3}z = 0 \\ \frac{2}{3}x + \frac{2}{3}y + \frac{2}{3}z = 0 \\ \frac{2}{3}x + \frac{2}{3}y + \frac{2}{3}z = 0 \end{cases}$$

We obtain $z = -x - y$, hence the eigenspace is:

$$S_{\lambda_1} = \{(\alpha, \beta, -\alpha - \beta), \alpha, \beta \in R\}.$$

The eigenvectors corresponding to λ_1 are $(\alpha, \beta, -\alpha - \beta), \alpha, \beta \in R, \quad (\alpha, \beta) \neq (0,0)$.

By proceeding similarly for $\lambda_2 = 1$ one obtains the corresponding eigenspace:

$$S_{\lambda_2} = \{(\alpha, \alpha, \alpha), \alpha \in R\}.$$

The eigenvectors corresponding to λ_2 are $(\alpha, \alpha, \alpha), \alpha \in R^*$.

10. Solve the integral equation

$$\int_0^\infty f(t) \cos ut \, dt = \begin{cases} 1 - u, u \in [0,1] \\ 0, u > 1. \end{cases}$$

Answer: The solution of the integral equation is:

$$f(t) = \frac{2}{\pi} \int_0^\infty g(u) \cos ut \, du = \frac{2}{\pi} \int_0^1 (1 - u) \cos ut \, du\,.$$

Integrating by parts, we obtain:

$$f(t) = \frac{2}{\pi}(1-u)\left.\frac{\sin ut}{t}\right|_0^1 + \frac{2}{\pi t}\int_0^1 \sin ut \, du$$

i.e.:

$$f(t) = \frac{2}{\pi t}\left.\frac{-\cos ut}{t}\right|_0^1 = \frac{2}{\pi t^2}(1-\cos t).$$

Physics

1. The Newton's laws

Answer

The first Newton's law

An object will remain at rest or in uniform linear motion unless acted upon by an external force (net force).

The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called ***inertia****, and the first law is sometimes called* ***the law of inertia****.*

The second Newton's law

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Newton considered the product of mass and velocity as a measure of an object's "quantity of motion." Today, we call the product of a particle's mass and velocity linear momentum: $\vec{p} = m\vec{v}$.

If the mass m of the particle is constant: $\vec{F} = m\vec{a}$.

The third Newton's law

"To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts".

$$\vec{F}_{12} = -\vec{F}_{21}$$

$[F]_{SI}$=N (Newton), $[p]_{SI}$=N·s (Newton·second), $[m]_{SI}$=kg (kilogram), $[v]_{SI}$=m/s (meter/second), $[a]_{SI}$=m/s^2.

2. The law of conservation of linear momentum

Answer – The total linear momentum of a system is conserved (remains constant) when the total external force acting on the system is zero or the system is a completely isolated system.

$$\vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \quad \Rightarrow \quad \vec{p} = \text{constant}$$

where $\vec{p}$– is the linear momentum , $\vec{F}$ – the net force, t – the time. The net force acting on a system is related to the rate of change of momentum of the system by Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}$.

$[F]_{SI}$ =N (Newton), $[p]_{SI}$=N·s (Newton·second).

3. Newton's universal law of gravitation

Answer

The force of gravitational attraction between two bodies is proportional to the product of their masses (M, m) and inversely proportional to the square of the distance r between their centers.

$$\vec{F} = -K\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

where K is the universal gravitational constant;

The gravitational acceleration or the gravitational field strength (gravitational field intensity) of a body with mass M at a distance r from its center is $\vec{g} = -K\frac{M}{r^2}\frac{\vec{r}}{r}$.

4. Simple Harmonic Oscillator (SHO) - motion law and total energy

Answer

Simple harmonic motion occurs whenever the restoring force is proportional to the displacement from equilibrium $F = -k_{el}x$.

Motion law for a SHO is the solution of second Newton's law for a net force (acting on a particle with mass m) equal with the restoring force.

$$x(t) = A\sin(\omega_0 t + \varphi)$$

where $\omega_0 = \sqrt{\frac{k_{el}}{m}}$ is the (natural) angular frequency for the SHO; The oscillations period is $T_0 = \frac{2\pi}{\omega_0}$ and the frequency $\nu_0 = 2\pi\omega_0$

The restoring force $F = -k_{el}x$ is a conservative force and thus the total energy for a SHO is conserved.

$$E = \frac{k_{el}A^2}{2}$$

$[F]_{SI}$ =N (Newton), $[k_{el}]_{SI}$=N/m (Newton/meter), $[m]_{SI}$=kg (kilogram), $[\omega_0]_{SI}$=radian/s.

5. Define the work

Answer

By definition, the elementary work (δW) done on a particle is the dot product between the force ($\vec{F}$) and the distance ($d\vec{r}$) through the particle moves.

$\delta W = \vec{F}\cdot d\vec{r}$, $\delta W = |\vec{F}|\cdot|d\vec{r}|\cdot cos\measuredangle(\vec{F}, d\vec{r})$, $W = \int_1^2 \vec{F}\cdot d\vec{r}$.

If the force $\vec{F}$ is constant on the entire displacement $\Delta\vec{r}$ the work on that displacement is $W = \vec{F}\cdot\Delta\vec{r}$.

The perpendicular component (on the path) of $\vec{F}$ does not make work

When the work done by the force $\vec{F}$ does not depend on the path (contained in $d\vec{r}$), we say that the force $\vec{F}$ is conservative.

$[F]_{SI}$=N (Newton), $[W]_{SI}$ = J (Joule).

6. The first and the second thermodynamics laws

Answer

The first thermodynamics law

The variation of the internal energy of a thermodynamic system equals the energy exchanged by the system with the surroundings:

$$dU = \delta Q - \delta W$$

δQ infinitesimal amount of heat , δW elementary work.

Sign convention:

L< 0 work done on the system by external bodies (or environment),

L>0 work performed by the system on external bodies (the environment);

Q>0 heat transferred to the system, received from the environment,

Q< 0 heat transferred to the environment from the system.

$[U]_{SI} = [Q]_{SI} = [W]_{SI}$ = J (Joule).

The second thermodynamics law -Kelvin and Clausius statement

One form of the second thermodynamics law, due to lord **Kelvin** (1851), states that:

N**o** cyclic process is possible whose sole result is a flow of heat from a single reservoir and the performance of equivalent work.

In other words, is quite impossible to transform the entire received heat into work.

The thermal efficiency, η, is the ratio between the work, W, done by the working medium in the direct cycle (the system does positive work), and the sum Q_{abs} of all the amounts of heat transferred to the working medium during one cycle by the heat sources.

$$\eta = \frac{W}{Q_{abs}}$$

The form of the second thermodynamics law, due to **Clausius** (1854), states that:
Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.
In other words, it is not possible for heat to flow from a colder body to a warmer body without any work having been done to accomplish this flow.
$[Q]_{SI} = [W]_{SI} = J$ (Joule).

7. The Ohm's law

Answer

Microscopic Ohm's law: The current density is proportional to the electric field, and the material-dependent parameter of proportionality is called conductivity (the reciprocal of resistivity $\sigma = \frac{1}{\rho}$).

$$\vec{j} = \sigma\vec{E}$$

The current density ($\vec{j}$) is the vector that points in the direction of the flow of positive charges having the magnitude equal to the amount of charge passing in unit time through unit area normal to the flow (the rate at which charges flow across the unit cross-sectional area).
$[j]_{SI}=A/m^2$, $[E]_{SI}=V/m$, $[\rho]_{SI}=\Omega m$, $[\sigma]_{SI}=1/(\Omega m)$.

A material that obeys the microscopic Ohm's law is said to be ohmic; otherwise, the material is non-ohmic. Most metals, with good conductivity and low resistivity, are ohmic.
Suppose a potential difference **U** applied between the ends of a wire of length ***l*** and cross-sectional area ***A*** , creating a constant electric field $\vec{\mathbf{E}}$ and the steady (direct) current (DC) **I**.

$$I = \frac{U}{R}$$

The "macroscopic" version of the Ohm's law : The current I through a conductor having the resistance R between two points is directly proportional to the potential difference (voltage) U across the two points, and depend on the resistance R related to resistivity by: $R = \rho\frac{l}{A}$.
$[I]_{SI}=A$ (Ampère), $[U]_{SI}=V$ (Volt), $[R]_{SI}=\Omega$ (Ohm).

8. Faraday' law of induction

Answer:

The rate of change of the magnetic flux give an electromotive force (emf). This is the basic principle of making voltage sources because if you have a varying magnetic flux, then you've got a voltage source.

$$\mathcal{E} = \oint_{\Gamma} \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_m}{\partial t}$$

The induced emf along a closed contour Γ is equal to the negative of the time rate of change of the magnetic flux through a surface Σ whose boundary is the curve Γ.
$\boldsymbol{\mathcal{E}}$ is the electromotive force (emf) induced in the circuit defined by the curve Γ,
Φ_m the magnetic flux through an arbitrary open surface Σ whose boundary is the curve Γ,

$$\Phi_m = \iint_{\Sigma} \vec{B} \cdot d\vec{S}$$

$\vec{B}$ is the magnetic field (the magnetic flux density).
$[\Phi_m]_{SI}$=Wb (Weber), $[\mathcal{E}]_{SI}$=V (Volt), $[B]_{SI}$=T (Tesla).

9. Waves attenuation law–application : Passing 3 cm in a material the intensity of a planar monochromatic wave decreases n_1 =1.5 times. How deep in the material the wave intensity will be n_2 =2.25 times smaller than at the incidence?

Answer

When a wave travels through a medium, its intensity diminishes with distance:

$$I = I_0 e^{-\alpha x}$$

I_0 = the intensity of the incident wave (the unattenuated intensity of the propagating wave at some location)
I = the intensity of the transmitted wave (the reduced intensity after the wave has traveled a distance **x** from that initial location)
α = the linear attenuation coefficient dependent upon the type of material, type of wave and the energy wave (wavelength).
$[I]_{SI}$=W/m^2 (Watt/ meter^2), $[\alpha]_{SI}$=m^{-1}(meter), $[x]_{SI}$=m.

$$\begin{cases} I_1 = I_0 e^{-\alpha x_1} \\ I_2 = I_0 e^{-\alpha x_2} \end{cases}$$

$$\Rightarrow \begin{cases} \ln n_1 = \alpha x_1 \\ \ln n_2 = \alpha x_2 \end{cases} \qquad \Rightarrow x_2 = x_1 \frac{\ln n_2}{\ln n_1} = 6\text{cm}$$

10. Electromagnetic force on an electric point charge –application: Electrons emited by a filament are accelerated from rest through a constant potential difference of 125V and then pass undeflected through a region with constant electric field of 2000V/m and a crossed magnetic field of 0.3mT (electric and magnetic field are perpendicular to each). Find the specific charge of electrons e/m_e.

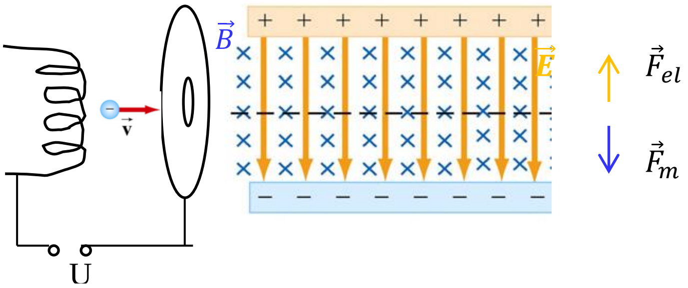
Answer

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

A particle of charge q in an electric field $\vec{E}$ experiences a force proportional with the electric field: $\vec{F} = q\,\vec{E}$. The electric force is exerted in the direction of the surrounding electric field.

A particle of charge q moving through a magnetic field $\vec{B}$ with the velocity $\vec{v}$ experiences a magnetic force - the Lorentz force: $\vec{F} = q\,\vec{v} \times \vec{B}$**.**

The magnetic force on a moving charge is exerted in a direction at a right angle to the plane formed by the direction of its velocity and the direction of the surrounding magnetic field.
$[q]_{SI}$=C (Coulomb), $[E]_{SI}$=V/m (Volt/m), $[B]_{SI}$=T (Tesla), $[F]_{SI}$=N (Newton), $[v]_{SI}$=m/s (meter/second).



The electrons with charge q= - e and mass m_e emitted from the filament are accelerated (from rest) by the potential difference U and receive a kinetic energy (W=ΔE_k – work kinetic energy theorem; W= - qU-work done on an electric charge by a uniform electric field)

$$eU = \frac{m_e v^2}{2}$$

The electrons enter then into the region with magnetic field ⊥ to the electric field and pass it undeflected if the net force is zero ($\vec{F} = 0 \Leftrightarrow F_{el} = F_m$).

Therefore, only those particles with speed **v=E/B** will be able to move in a straight line.

$$\Rightarrow \frac{e}{m_e} = \frac{E^2}{2UB^2} \qquad \Rightarrow \frac{e}{m_e} = 1.777 \cdot 10^{11} C/kg$$

Obs. The 2014 CODATA (Committee on Data for Science and Technology) recommended the values:

$$e = 1.60217662 \cdot 10^{-19} C$$
$$m_e = 9.10938356 \cdot 10^{-31} kg$$
$$\frac{e}{m_e} = 1.758820024 \cdot 10^{11} C/kg$$

MEASUREMENT UNITS

1. Specify the SI unit and its symbol for time. Specify the multiplier and its symbol for micro (example: atto = 10^{-18}, a).
   The SI unit for time is the second. Its symbol is s. The multiplier for micro is 10^{-6}. Its symbol is μ.
2. Specify the SI unit and its symbol for electrical current. Specify the multiplier and its symbol for milli (example: atto = 10^{-18}, a).
   The SI unit for electrical current is the ampere. Its symbol is A. The multiplier for milli is 10^{-3}. Its symbol is m.
3. Specify the SI unit and its symbol for frequency. Specify the multiplier and its symbol for tera (example: atto = 10^{-18}, a).
   The SI unit for frequency is the hertz. Its symbol is Hz. The multiplier for tera is 10^{12}. Its symbol is T.
4. Specify the SI unit and its symbol for power and radiant flux. Specify the multiplier and its symbol for giga (example: atto = 10^{-18}, a).
   The SI unit for power and radiant flux is the watt. Its symbol is W. The multiplier for giga is 10^{9}. Its symbol is G.
5. Specify the SI unit and its symbol for electrical charge and quantity of electricity. Specify the multiplier and its symbol for nano (example: atto = 10^{-18}, a).
   The SI unit for electrical charge and quantity of electricity is the coulomb. Its symbol is C. The multiplier for nano is 10^{-9}. Its symbol is n.
6. Specify the SI unit and its symbol for voltage, electrical potential difference and electromotive force. Specify the multiplier and its symbol for kilo (example: atto = 10^{-18}, a).
   The SI unit for voltage, electrical potential difference and electromotive force is the volt. Its symbol is V. The multiplier for kilo is 10^{3}. Its symbol is k.
7. Specify the SI unit and its symbol for electric resistance, impedance and reactance. Specify the multiplier and its symbol for mega (example: atto = 10^{-18}, a).
   The SI unit for electric resistance, impedance and reactance is the ohm. Its symbol is Ω. The multiplier for mega is 10^{6}. Its symbol is M.
8. Specify the SI unit and its symbol for electric capacitance. Specify the multiplier and its symbol for pico (example: atto = 10^{-18}, a).
   The SI unit for electric capacitance is the farad. Its symbol is F. The multiplier for pico is 10^{-12}. Its symbol is p.
9. A resistor has a resistance of 0.00035 Ω. Convert it into mΩ and μΩ.
   $0.00035\ \Omega = 0.00035 \times 10^{3}\ m\Omega = 0.35\ m\Omega$
   $0.00035\ \Omega = 0.00035 \times 10^{6}\ \mu\Omega = 350\ \mu\Omega.$
10. Express in kHz and in MHz the frequency of a signal whose period is 20 μs.

$$f = \frac{1}{T} = \frac{1}{20\ \mu s} = \frac{1}{20 \times 10^{-6}\ s} = \frac{10^{6}}{20} Hz = 0.05\ MHz = 50\ kHz.$$