Subiecte de Analiză Matematică pentru examenul de licenţă la ETC-eng

1. State the Ratio test for the convergence of numerical series with positive terms.
   Solution:

Let $x_n, n \geq 0$, be a sequence with positive terms, $x_n > 0$. Assume the limit $l = \lim_{n\to\infty} \frac{x_{n+1}}{x_n}$ exists.

If $l < 1$, then the numerical series $\sum_{n\geq 0} x_n$ is convergent.

If $l > 1$, then the numerical series $\sum_{n\geq 0} x_n$ is divergent.

If $l = 1$, the Ratio test does not decide on the nature of the series $\sum_{n\geq 0} x_n$.

2. Let $f: A \to R$ be a function defined on an open set A from R^3, $f = f(x,y,z)$, and $u_0 = (x_0, y_0, z_0) \in A$ a fixed point from A. Define the partial derivatives $\frac{\partial f}{\partial x}(u_0)$, $\frac{\partial f}{\partial y}(u_0)$ and $\frac{\partial f}{\partial z}(u_0)$.
   Solution:

The partial derivatives are defined as it follows:

$$\frac{\partial f}{\partial x}(u_0) = \lim_{x\to x_0} \frac{f(x,y,z) - f(x_0,y_0,z_0)}{x - x_0},$$

$$\frac{\partial f}{\partial y}(u_0) = \lim_{y\to y_0} \frac{f(x,y,z) - f(x_0,y_0,z_0)}{y - y_0},$$

$$\frac{\partial f}{\partial z}(u_0) = \lim_{z\to z_0} \frac{f(x,y,z) - f(x_0,y_0,z_0)}{z - z_0}.$$

3. State the Schwarz theorem for a function f defined on an open set A from R^3, $f: A \to R$.
   Solution:

Let $f: A \to R$ be a function defined on an open set A from R^3, $f = f(x,y,z)$, and $u_0 = (x_0, y_0, z_0) \in A$ an arbitrary point from A. If f is a function of class C^2 on A, then $\frac{\partial^2 f}{\partial x \partial y}(u_0) = \frac{\partial^2 f}{\partial y \partial x}(u_0)$, $\frac{\partial^2 f}{\partial x \partial z}(u_0) = \frac{\partial^2 f}{\partial z \partial x}(u_0)$ and $\frac{\partial^2 f}{\partial y \partial z}(u_0) = \frac{\partial^2 f}{\partial z \partial y}(u_0)$.

4. Let $f: A \to R$ be a function defined on an open set A from R^2, $f = f(x,y)$, and $u_0 = (x_0, y_0) \in A$. Approximate the function $f(x,y)$ around the value $f(u_0)$ using the Taylor polynomial $T_2(x,y)$ of order two. Write the expression of $T_2(x,y)$.
   Solution:

$f(x,y) \approx T_2(x,y) = f(u_0) + (x-x_0)\frac{\partial f}{\partial x}(u_0) + (y-y_0)\frac{\partial f}{\partial y}(u_0) + \frac{1}{2}\Big[(x-x_0)^2\frac{\partial^2 f}{\partial x^2}(u_0) + 2(x-x_0)(y-y_0)\frac{\partial^2 f}{\partial x \partial y}(u_0) + (y-y_0)^2\frac{\partial^2 f}{\partial y^2}(u_0)\Big]$.

5. Find the extremum points of the function $f: R^2 \to R$, where $f(x,y) = x^2 - 2xy + \frac{2}{3}y^3$.
   Solution:

Find first the critical points $f(x,y) = x^2 - 2xy + \frac{2}{3}y^3$ from $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$. These equations lead to the system $\begin{cases} 2x - 2y = 0 \\ -2x + 2y^2 = 0 \end{cases}$ which has the solutions $u_1 = (0,0)$ and $u_2 = (1,1)$.

Method 1. The differential of order two is

$$d^2 f(h_1,h_2) = \frac{\partial^2 f}{\partial x^2}h_1^2 + 2\frac{\partial^2 f}{\partial x \partial y}h_1 h_2 + \frac{\partial^2 f}{\partial y^2}h_2^2,$$

where $h = (h_1,h_2) \neq (0,0)$, that is,

$$d^2 f(h_1,h_2) = 2h_1^2 - 4h_1h_2 + 4yh_2^2.$$

At $u_1 = (0,0)$ we have $d^2_{u_1} f(h) = 2h_1^2 - 4h_1h_2$, which is undefined, thus u_1 is a saddle point.

At $u_2 = (1,1)$ we have $d^2_{u_2} f(h) = 2(h_1 - h_2)^2 + 2h_2^2 > 0$ for all $(h_1,h_2) \neq (0,0)$, thus, u_2 is a local minimum point of the function f on its domain of definition.

Method 2. The Hessian of f at a general point (x,y) is

$$H = \begin{pmatrix} 2 & -2 \\ -2 & 4y \end{pmatrix}.$$

At $u_1 = (0,0)$ the Hessian becomes $H = \begin{pmatrix} 2 & -2 \\ -2 & 0 \end{pmatrix}$, which has the determinants

$$\Delta_1 = 2 > 0 \text{ and } \Delta_2 = \begin{vmatrix} 2 & -2 \\ -2 & 0 \end{vmatrix} = -4 < 0, \text{ thus, } u_1 \text{ is a saddle point.}$$

At $u_2 = (1,1)$ the Hessian becomes $H = \begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix}$, which has the determinants

$$\Delta_1 = 2 > 0 \text{ and } \Delta_2 = \begin{vmatrix} 2 & -2 \\ -2 & 4 \end{vmatrix} = 4 > 0, \text{ thus, } u_2 \text{ is a local minimum point.}$$

6. Apply the Implicit Function Theorem to the equation $x^2 - xy + 2y^3 = 0$ to find a function $y = y(x)$ for $x \in V_1$, where V_1 is a neighborhood of $x = 1$. Writing $y(x) \approx a + bx$, find a and b.
   Solution:

Consider the function $F(x,y) = x^2 - xy + 2y^3$. Since $F(1,-1) = 0$ and $\frac{\partial F}{\partial y}(x,y) = -x + 6y^2$ with $\frac{\partial F}{\partial y}(1,-1) = 5 \neq 0$, the Implicit Function Theorem can be applied. Thus, there exist an open neighborhood V_1 of $x = 1$ and an open neighborhood V_{-1} of $y = -1$, and a function $y: V_1 \to V_{-1}$ of class C^1 on V_1, $y = y(x)$, such that:

$$y(1) = -1 \text{ and } F\big(x, y(x)\big) = 0 \text{ for all } x \in V_1,$$

respectively,

$$y'(x) = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{2x - y}{-x + 6y^2},$$

and $y'(1) = -\frac{3}{5}$.

Using the Taylor approximation of the function $y(x)$ at $x = 1$, we have

$$y(x) \approx y(1) + (x-1)y'(1) = -1 - \tfrac{3}{5}(x-1) = -\tfrac{2}{5} - \tfrac{3}{5}x,$$

thus, $a = -\frac{2}{5}$ and $b = -\frac{3}{5}$.

7.

a. Define the orthonormal basis of an euclidean vector space.
b. Give an example of orthonormal basis of $\mathbf{R}^3$. Argue why it is such a basis.

Proof:

a. A basis of an euclidean vector space that consists of vectors orthogonal to one another, is called an orthogonal basis. We call an orthogonal basis such that its vectors are of norm equal to 1, an orthonormal basis.
b. The canonical basis

$$\mathrm{B_c} = \{(1,0,0), (0,1,0), (0,0,1)\}$$

is an orthonormal basis of $\mathbf{R}^3$. The vectors of these basis are orthogonal to one another because

$$(1,0,0) \cdot (0,1,0) = 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0;$$

$$(1,0,0) \cdot (0,0,1) = 0;$$

$$(0,1,0) \cdot (0,0,1) = 0.$$

The vectors of the basis are of norm equal to 1. Indeed

$$\|(1,0,0)\| = \sqrt{1^2 + 0^2 + 0^2} = \|(0,1,0)\| = \|(0,0,1)\|.$$

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8. We consider the linear mapping $\mathrm{L}: \mathrm{R}^3 \to \mathrm{R}^3$ such that the matrix associated to L with respect to the canonical basis $B_c = \{(1,0,0), (0,1,0), (0,0,1)\}$ is

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}.$$

Find the kernel Ker L. Check whether the mapping L is injective.

Proof:

Because $\det \mathrm{A} = 1 \neq 0$, it follows that

$$\operatorname{rank} \mathrm{A} = 3.$$

We have

$$\mathrm{Ker} L = \left\{ (x_1, x_2, x_3) \middle| A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\} =$$

$$= \{(x_1, x_2, x_3) \mid (x_1 = 0, x_2 = 0, -x_2 + x_3 = 0\}.$$

We get

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ -x_2 + x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

Hence

$$\mathrm{Ker} L = \{(0,0,0)\}.$$

The mapping L is therefore injective.

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9.

a. Define the cross product of two nonzero vectors of $\mathbf{R}^3$.

b. Give the formula of the length of the cross product of two nonzero vectors of $\mathbf{R}^3$. What does this value represent?

Proof:

Let

$$\overline{u} = u_1 \overline{i} + u_2 \overline{j} + u_3 \overline{k}$$

and

$$\overline{v} = v_1 \overline{i} + v_2 \overline{j} + v_3 \overline{k}$$

be vectors of $\mathbf{R}^3$.

a. The cross product $\overline{u} \times \overline{v}$ is the vector

$$\overline{u} \times \overline{v} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

b. We compute the length of the cross product of $\overline{u}$ and $\overline{v}$ by the formula

$$\left|\left|\overline{u} \times \overline{v}\right|\right| = \left|\left|\overline{u}\right|\right| \cdot \left|\left|\overline{v}\right|\right| \cdot \sin\left(\angle(\overline{u}, \overline{v})\right).$$

The length of the cross product of two nonzero vectors represents the area of the parallelogram built on the two vectors.

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10. Find the orthogonal projection N of the point $M(1,0,1)$ onto the plan

$$\pi: x - y + z = 0.$$

Then find the distance MN.

Proof:

Note that the normal vector of the plane π is $\overline{n}(1,-1,1)$. We consider the line d passing through the point $M(1,0,1)$ which is perpendicular to π. Then $\overline{d} = \overline{n}(1,-1,1)$. Therefore the canonical equations of the line d are:

$$d: \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1} = t.$$

The parametric equations of the line d are

$$d: \begin{cases} x = 1 + t, \\ y = -t, \\ z = 1 + t. \end{cases}$$

We find the intersection point $N = d \cap \pi$:

$$1 + t_0 + t_0 + 1 + t_0 = 0.$$

Solving this equation, we get $t_0 = -\frac{2}{3}$.

Then

$$N\left(1 - \frac{2}{3}, -\left(-\frac{2}{3}\right), 1 - \frac{2}{3}\right) = N\left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

is the orthogonal projection of the point M onto the plane π.

Note that

$$MN = d(M, \pi) = \frac{|x_M - y_M + z_M|}{\sqrt{1 + (-1)^2 + 1}} = \frac{|1 - 0 + 1|}{\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

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PHYSICS

1. The Newton's laws

Answer

The first Newton's law

An object will remain at rest or in uniform linear motion unless acted upon by an external force (net force).

*The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called **inertia**, and the first law is sometimes called **the law of inertia**.*

The second Newton's law

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of the force.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Newton considered the product of mass and velocity as a measure of an object's "quantity of motion." Today, we call the product of a particle's mass and velocity linear momentum: $\vec{p} = m\vec{v}$.

If the mass m of the particle is constant: $\vec{F} = m\vec{a}$.

The third Newton's law

"To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts".

$$\vec{F}_{12} = -\vec{F}_{21}$$

$[F]_{SI}$=N (Newton), $[p]_{SI}$=N·s (Newton·second), $[m]_{SI}$=kg (kilogram), $[v]_{SI}$=m/s (meter/second), $[a]_{SI}$=m/s^2.

2. The law of conservation of linear momentum

Answer – The total linear momentum of a system is conserved (remains constant) when the total external force acting on the system is zero or the system is a completely isolated system.

$$\vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \quad \Rightarrow \quad \vec{p} = \text{constant}$$

where $\vec{p}$– is the linear momentum , $\vec{F}$ – the net force, t – the time. The net force acting on a system is related to the rate of change of momentum of the system by Newton's second law:

$\vec{F} = \frac{d\vec{p}}{dt}$.

$[F]_{SI}$ =N (Newton), $[p]_{SI}$=N·s (Newton·second).

3. Newton's universal law of gravitation

Answer

The force of gravitational attraction between two bodies is proportional to the product of their masses (M, m) and inversely proportional to the square of the distance r between their centers.

$$\vec{F} = -K\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

where K is the universal gravitational constant;

The gravitational acceleration or the gravitational field strength (gravitational field intensity)

of a body with mass M at a distance r from its center is $\vec{g} = -K\frac{M}{r^2}\frac{\vec{r}}{r}$.

4. Simple Harmonic Oscillator (SHO) - motion law and total energy

Answer

Simple harmonic motion occurs whenever the restoring force is proportional to the displacement from equilibrium $F = -k_{el}x$.

Motion law for a SHO is the solution of second Newton's law for a net force (acting on a particle with mass m) equal with the restoring force.

$$x(t) = A\sin(\omega_0 t + \varphi)$$

where $\omega_0 = \sqrt{\frac{k_{el}}{m}}$ is the (natural) angular frequency for the SHO; The oscillations period is $T_0 = \frac{2\pi}{\omega_0}$ and the frequency $\nu_0 = 2\pi\omega_0$

The restoring force $F = -k_{el}x$ is a conservative force and thus the total energy for a SHO is conserved.

$$E = \frac{k_{el}A^2}{2}$$

$[F]_{SI}$ =N (Newton), $[k_{el}]_{SI}$=N/m (Newton/meter), $[m]_{SI}$=kg (kilogram), $[\omega_0]_{SI}$=radian/s.

5. Define the work

Answer

By definition, the elementary work (δW) done on a particle is the dot product between the force ($\vec{F}$) and the distance ($d\vec{r}$) through the particle moves.

$$\delta W = \vec{F}\cdot d\vec{r}, \qquad \delta W = |\vec{F}|\cdot|d\vec{r}|\cdot cos\sphericalangle(\vec{F}, d\vec{r}), \qquad W = \int_1^2 \vec{F}\cdot d\vec{r}.$$

If the force $\vec{F}$ is constant on the entire displacement $\Delta\vec{r}$ the work on that displacement is $W = \vec{F}\cdot\Delta\vec{r}$.

The perpendicular component (on the path) of $\vec{F}$ does not make work

When the work done by the force $\vec{F}$ does not depend on the path (contained in $d\vec{r}$), we say that the force $\vec{F}$ is conservative.

$[F]_{SI}$=N (Newton), $[W]_{SI} = J$ (Joule).

6. The first and the second thermodynamics laws

Answer

The first thermodynamics law

The variation of the internal energy of a thermodynamic system equals the energy exchanged by the system with the surroundings:

$$\mathrm{dU} = \delta\mathrm{Q} - \delta\mathrm{W}$$

$\delta\mathrm{Q}$ infinitesimal amount of heat , $\delta\mathrm{W}$ elementary work.

Sign convention:

L< 0 work done on the system by external bodies (or environment),

L>0 work performed by the system on external bodies (the environment);

Q>0 heat transferred to the system, received from the environment,

Q< 0 heat transferred to the environment from the system.

$[\mathrm{U}]_{\mathrm{SI}} = [\mathrm{Q}]_{\mathrm{SI}} = [W]_{\mathrm{SI}} = \mathrm{J}$ (Joule).

The second thermodynamics law -Kelvin and Clausius statement

One form of the second thermodynamics law, due to lord **Kelvin** (1851), states that:

N**o** cyclic process is possible whose sole result is a flow of heat from a single reservoir and the performance of equivalent work.

In other words, is quite impossible to transform the entire received heat into work.

The thermal efficiency**,** η, is the ratio between the work, W, done by the working medium in the direct cycle (the system does positive work), and the sum Q_{abs} of all the amounts of heat transferred to the working medium during one cycle by the heat sources.

$$\eta = \frac{W}{Q_{abs}}$$

The form of the second thermodynamics law, due to **Clausius** (1854), states that:

Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

In other words, it is not possible for heat to flow from a colder body to a warmer body without any work having been done to accomplish this flow.

$[\mathrm{Q}]_{\mathrm{SI}} = [W]_{\mathrm{SI}} = \mathrm{J}$ (Joule).

7. The Ohm's law

Answer

Microscopic Ohm's law: The current density is proportional to the electric field, and the material-dependent parameter of proportionality is called conductivity (the reciprocal of resistivity $\sigma = \frac{1}{\rho}$).

$$\vec{j} = \sigma\vec{E}$$

The current density ($\vec{j}$) is the vector that points in the direction of the flow of positive charges having the magnitude equal to the amount of charge passing in unit time through unit area normal to the flow (the rate at which charges flow across the unit cross-sectional area).

$[j]_{SI}=A/m^2$, $[E]_{SI}=V/m$, $[\rho]_{SI}=\Omega m$, $[\sigma]_{SI}=1/(\Omega m)$.

A material that obeys the microscopic Ohm's law is said to be ohmic; otherwise, the material is non-ohmic. Most metals, with good conductivity and low resistivity, are ohmic.

Suppose a potential difference **U** applied between the ends of a wire of length ***l*** and cross-sectional area ***A*** , creating a constant electric field $\vec{\mathbf{E}}$ and the steady (direct) current (DC) **I**.

$$I = \frac{U}{R}$$

The "macroscopic" version of the Ohm's law : The current I through a conductor having the resistance R between two points is directly proportional to the potential difference (voltage) U across the two points, and depend on the resistance R related to resistivity by: $R = \rho \frac{l}{A}$.

$[I]_{SI}=A$ (Ampère), $[U]_{SI}=V$ (Volt), $[R]_{SI}=\Omega$ (Ohm).

8. Faraday' law of induction

Answer:

The rate of change of the magnetic flux give an electromotive force (emf). This is the basic principle of making voltage sources because if you have a varying magnetic flux, then you've got a voltage source.

$$\mathcal{E} = \oint_{\Gamma} \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{l}} = -\frac{\partial \Phi_{\mathrm{m}}}{\partial \mathrm{t}}$$

The induced emf along a closed contour Γ is equal to the negative of the time rate of change of the magnetic flux through a surface Σ whose boundary is the curve Γ.
$\boldsymbol{\mathcal{E}}$ is the electromotive force (emf) induced in the circuit defined by the curve Γ,

Φ_m the magnetic flux through an arbitrary open surface Σ whose boundary is the curve Γ,

$$\Phi_{\mathrm{m}} = \iint_{\Sigma} \vec{\mathrm{B}} \cdot \mathrm{d}\vec{\mathrm{S}}$$

$\vec{\mathrm{B}}$ is the magnetic field (the magnetic flux density).

$[\Phi_m]_{SI}=Wb$ (Weber), $[\mathcal{E}]_{SI}=V$ (Volt), $[B]_{SI}=T$ (Tesla).

9. Waves attenuation law–application : Passing 3 cm in a material the intensity of a planar monochromatic wave decreases n_1 =1.5 times. How deep in the material the wave intensity will be n_2 =2.25 times smaller than at the incidence?

Answer

When a wave travels through a medium, its intensity diminishes with distance:

$$\mathrm{I} = \mathrm{I}_0 \mathrm{e}^{-\alpha\, \mathrm{x}}$$

I_0 = the intensity of the incident wave (the unattenuated intensity of the propagating wave at some location)

I = the intensity of the transmitted wave (the reduced intensity after the wave has traveled a distance **x** from that initial location)

α = the linear attenuation coefficient dependent upon the type of material, type of wave and the energy wave (wavelength).

$[I]_{SI}$=W/m^2 (Watt/ $meter^2$), $[\alpha]_{SI}$=m^{-1}(meter), $[x]_{SI}$=m.

$$\begin{cases} I_1 = I_0 e^{-\alpha x_1} \\ I_2 = I_0 e^{-\alpha x_2} \end{cases}$$

$$\Rightarrow \begin{cases} \ln n_1 = \alpha x_1 \\ \ln n_2 = \alpha x_2 \end{cases} \qquad \Rightarrow x_2 = x_1 \frac{\ln n_2}{\ln n_1} = 6\text{cm}$$

10. Electromagnetic force on an electric point charge –application: Electrons emited by a filament are accelerated from rest through a constant potential difference of 125V and then pass undeflected through a region with constant electric field of 2000V/m and a crossed magnetic field of 0.3mT (electric and magnetic field are perpendicular to each). Find the specific charge of electrons e/m_e.

Answer

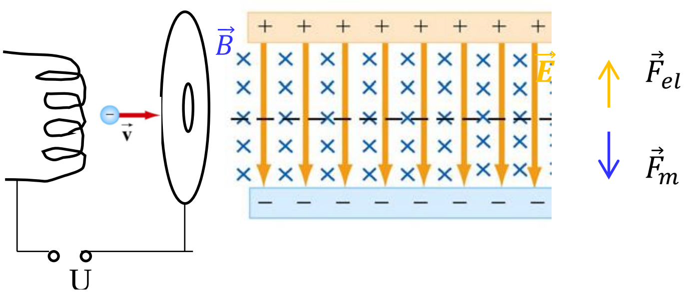
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

A particle of charge q in an electric field $\vec{E}$ experiences a force proportional with the electric field: $\vec{F} = q\,\vec{E}$. The electric force is exerted in the direction of the surrounding electric field.

A particle of charge q moving through a magnetic field $\vec{B}$ with the velocity $\vec{v}$ experiences a magnetic force - the Lorentz force: $\vec{F} = q\,\vec{v} \times \vec{B}$**.**

The magnetic force on a moving charge is exerted in a direction at a right angle to the plane formed by the direction of its velocity and the direction of the surrounding magnetic field.

$[q]_{SI}$=C (Coulomb), $[E]_{SI}$=V/m (Volt/m), $[B]_{SI}$=T (Tesla), $[F]_{SI}$=N (Newton), $[v]_{SI}$=m/s (meter/second).



The electrons with charge q= - e and mass m_e emitted from the filament are accelerated (from rest) by the potential difference U and receive a kinetic energy (W=ΔE_k – work kinetic energy theorem; W= - qU-work done on an electric charge by a uniform electric field)

$$eU = \frac{m_e v^2}{2}$$

The electrons enter then into the region with magnetic field $\perp$ to the electric field and pass it undeflected if the net force is zero ($\vec{F} = 0 \ \Leftrightarrow F_{el} = F_m$).

Therefore, only those particles with speed **v=E/B** will be able to move in a straight line.

$$\Rightarrow \frac{e}{m_e} = \frac{E^2}{2UB^2} \qquad\qquad \Rightarrow \frac{e}{m_e} = 1.777 \cdot 10^{11} C/kg$$

Obs. The 2014 CODATA (Committee on Data for Science and Technology) recommended the values:

$$e = 1.60217662 \ \cdot 10^{-19} C$$

$$m_e = 9.10938356 \ \cdot 10^{-31} kg$$

$$\frac{e}{m_e} = 1.758820024 \cdot 10^{11} C/kg$$

MEASUREMENT UNITS

1. Specify the SI unit and its symbol for time. Specify the multiplier and its symbol for micro (example: atto = 10^{-18}, a).
   The SI unit for time is the second. Its symbol is s. The multiplier for micro is 10^{-6}. Its symbol is μ.

2. Specify the SI unit and its symbol for electrical current. Specify the multiplier and its symbol for milli (example: atto = 10^{-18}, a).
   The SI unit for electrical current is the ampere. Its symbol is A. The multiplier for milli is 10^{-3}. Its symbol is m.

3. Specify the SI unit and its symbol for frequency. Specify the multiplier and its symbol for giga (example: atto = 10^{-18}, a).
   The SI unit for frequency is the hertz. Its symbol is Hz. The multiplier for tera is 10^{9}. Its symbol is G.

4. Specify the SI unit and its symbol for power. Specify the multiplier and its symbol for mega (example: atto = 10^{-18}, a).
   The SI unit for power is the watt. Its symbol is W. The multiplier for mega is 10^{6}. Its symbol is M.

5. Specify the SI unit and its symbol for inductance. Specify the multiplier and its symbol for nano (example: atto = 10^{-18}, a).
   The SI unit for inductance is the henry. Its symbol is H. The multiplier for nano is 10^{-9}. Its symbol is n.

6. Specify the SI unit and its symbol for voltage, electrical potential difference and electromotive force. Specify the multiplier and its symbol for kilo (example: atto = 10^{-18}, a).

   The SI unit for voltage, electrical potential difference and electromotive force is the volt. Its symbol is V. The multiplier for kilo is 10^{3}*. Its symbol is k.*

7. Specify the SI unit and its symbol for electric resistance, impedance and reactance. Specify the multiplier and its symbol for mega (example: atto = 10^{-18}, a).

   The SI unit for electric resistance, impedance and reactance is the ohm. Its symbol is Ω*. The multiplier for mega is* 10^{6}*. Its symbol is M.*

8. Specify the SI unit and its symbol for electric capacitance. Specify the multiplier and its symbol for pico (example: atto = 10^{-18}, a).

   The SI unit for electric capacitance is the farad. Its symbol is F. The multiplier for pico is 10^{-12}*. Its symbol is p.*

9. A current has a measured value of 0.00035 A. Convert it into mA and µA.

   $0.00035\ A=0.00035\times10^{3}\ mA=0.35\ mA$

   $0.00035\ A=0.00035\times10^{6}\ \mu A=350\ \mu A.$

10. Express in kHz and in MHz the frequency of a signal whose period is 20 µs.

$$f=\frac{1}{T}=\frac{1}{20\ \mu s}=\frac{1}{20\times10^{-6}\ s}=\frac{10^{6}}{20}Hz=0.05\ MHz=50\ kHz.$$